TRINITY COLLEGE

Year 12 Methods Units 3,4
Test 1 2019

Section 1 Calculator Free

Differentiation, Applications of Differentiation, Integration, Applications of Differentiation

STUDENT'S NAME SOLUTIONS

DATE: Friday 8th March **TIME:** 20 minutes **MARKS:** 21

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

1. (4 marks)

Determine each of the following

(a) $\int \frac{2-x^5}{x^3} dx$ $= \int 2x^{-3} - x^2 \, dx$ [2]

$= \frac{2x^{-2}}{-2} - \frac{x^3}{3} + c$

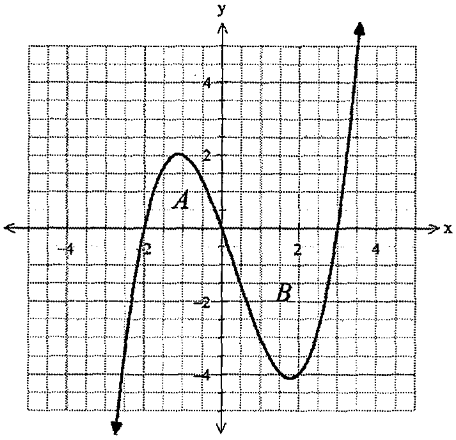
(b) $\int_1^4 (2x+3)dx$ $= \left[x^2 + 3x \right]_1^4$ [2]

$= (16+12) - (1+3)$

$= 24$

2. (9 marks)

Given the graph of $y = f(x)$ below where area A = 7 cm^2 and area B = 18 cm^2



(a) Determine

(i) $\int_{-2}^{3} f(x)dx$ -11 [1]

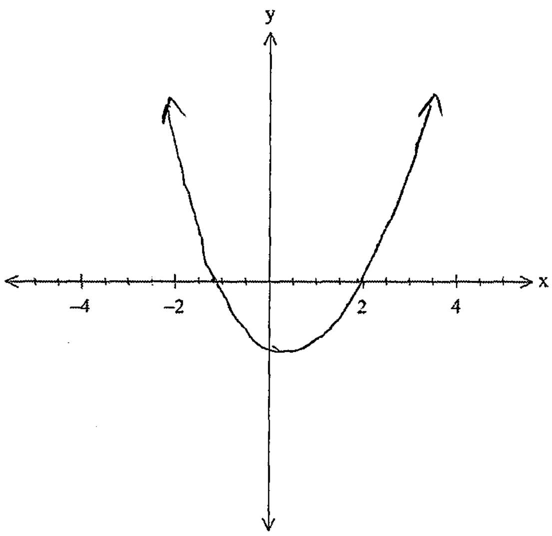
(ii) $\int_{-2}^{3} |f(x)| dx$ 25 [1]

(iii) $\int_{-2}^{3} -f(x)dx$ 11 [1]

(iv) $\int_{-2}^{3} (f(x)+2)dx$ [3]

$$= \int_{-2}^{3} f(x)\,dx + \int 2\,dx$$
$$= -11 + [2x]_{-2}^{3}$$
$$= -11 + 6 - (-4)$$
$$= -1$$

(b) Sketch $y = f'(x)$ [2]



(c) Using your graph, determine when $f''(x) < 0$ [1]

$x \approx < 0.5$

3. (4 marks)

The gradient at any point on a curve is given by $\frac{dy}{dx} = \frac{1}{\sqrt{4-3x}}$. Determine the equation of the curve that passes through the point (-4,3).

$$y = \int (4-3x)^{-\frac{1}{2}}\, dx$$

$$y = \frac{(4-3x)^{\frac{1}{2}}}{\frac{1}{2}(-3)} + c$$

$(-4,3)$ $$3 = -\frac{2}{3}(16)^{\frac{1}{2}} + c$$

$$\frac{17}{3} = c$$

$$y = -\frac{2}{3}(4-3x)^{\frac{1}{2}} + \frac{17}{3}$$

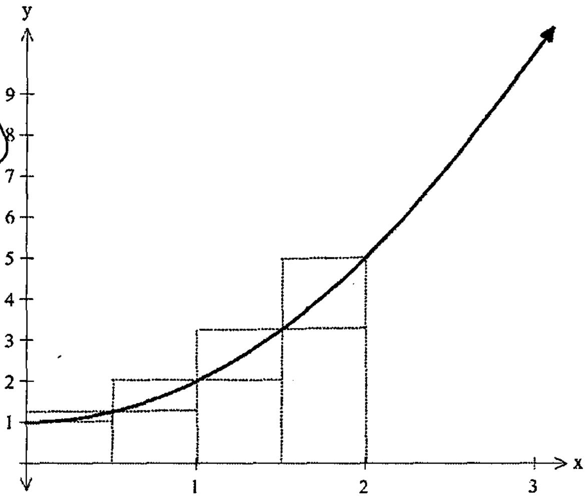
4. (4 marks)

Given the function $y = x^2 + 1$

(a) Complete the table below. [1]

0	0.5	1	1.5	2
1	1.25	2	2.25	5

(b) Calculate an underestimate of the area under the function for $0 \le x \le 2$ using 4 rectangles. [2]



$\text{AREA} = \left(\frac{1}{2} \times 1\right) + \left(\frac{1}{2} \times 1.25\right) + \left(\frac{1}{2} \times 2\right) + \left(\frac{1}{2} \times 2.25\right)$

$= \frac{1}{2}\left(1 + 1.25 + 2 + 2.25\right)$

$= 3.25$

(c) The overestimate of the area under the function for $0 \le x \le 2$ is 5.25 using 4 rectangles.

Give a more accurate estimate of the area under the function for $0 \le x \le 2$ using 4 rectangles. [1]

$\dfrac{3.25 + 5.25}{2}$

$= 4.25$

TRINITY COLLEGE

Year 12 Methods Units 3,4
Test 1 2019

Section 2 Calculator Assumed

Differentiation, Applications of Differentiation, Integration, Applications of Integration

STUDENT'S NAME ______________________________

DATE: Friday 8th March **TIME:** 30 minutes **MARKS:** 32

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, formula sheets

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

5. (4 marks)

Newton's Law of Gravitation states that the force F of attraction between two particles having masses of m_1 and m_2 is given by $F = \frac{m_1 m_2 g}{s^2}$ where g is a constant and s is the distance between the two particles. If $s = 20\,cm$, use the increments formula to determine the approximate percentage change in s that will increase F by 8%.

$\delta F \approx \frac{dF}{ds} \times \delta s$

$\frac{\delta F}{F} \approx \frac{dF}{ds} \times \frac{\delta s}{F}$

$0.08 \approx \frac{-2\cancel{m_1 m_2 g}}{s^{\cancel{3}}} \times \frac{\delta s}{\cancel{m_1 m_2 g}}\cancel{s^2}$

$\frac{0.08}{-2} \approx \frac{\delta s}{s}$

$-0.04 \approx \frac{\delta s}{s}$

ie 4% DECREASE IN S

$\frac{\delta F}{F} = 0.08$

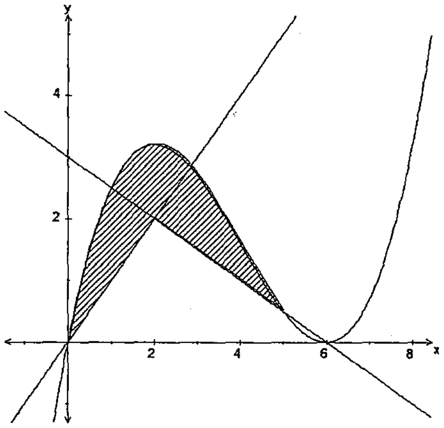
$\frac{\delta s}{s} = ?$

$F = m_1 m_2 g\, s^{-2}$

$\frac{dF}{ds} = m_1 m_2 g(-2)s^{-3}$

6. (6 marks)

A new shape is being proposed for the boomerang throwing event in the 2032 Olympics being held in Perth. The cross-section (shaded) is formed by the intersection of three curves as shown.



$0.1x(x-6)^2 = 3 - 0.5x$

$\therefore x = 5$

$x = 3 - 0.5x$

$x = 2$

The curves have equations $f(x) = 0.1x(x-6)^2$, $g(x) = x$ and $h(x) = 3 - 0.5x$. The scale used is in cm.

The boomerang is 3 mm thick and is made from a material which has a density of 8 g per cm^3. Calculate the weight of the boomerang.

AREA $= \int_0^5 0.1x(x-6)^2\,dx - \int_0^2 x\,dx - \int_2^5 3 - 0.5x\,dx$

$= 10.625 - 2 - 3.75$

$= 4.875$

WEIGHT $= 4.875 \times 0.3 \times 8$

$= 11.7$ gm

7. (4 marks)

The area enclosed by the curves $y = mx$ and $y = x^2$ is 24.813. Determine the value of m where $m > 0$.

$$\int_0^m mx - x^2 \, dx = 24.813$$

$$mx = x^2$$
$$0 = x^2 - mx$$
$$0 = x(x - m)$$
$$x = 0, m$$

$$\left[\frac{mx^2}{2} - \frac{x^3}{3} \right]_0^m = 24.813$$

$$\frac{m^3}{2} - \frac{m^3}{3} = 24.813$$

$$\frac{m^3}{6} = 24.813$$

$$m = 5.3$$

8. (10 marks)

A particle travels in a straight line. Its velocity as it passes through a fixed point O is 2 ms^{-1}. The acceleration, t seconds after passing O, is given by $a = 6t - 6\,\text{ms}^{-2}$. Calculate

(a) the velocity after 2 seconds. [3]

$v = \int 6t - 6 \; dt$

$v = 3t^2 - 6t + c$

$t = 0$
$x = 0$
$v = 2$

$2 = c$

$v = 3t^2 - 6t + 2$

$v(2) = 12 - 12 + 2$

$= 2$

(b) the maximum displacement for $0 \le t \le 2$. [3]

$3t^2 - 6t + 2 = 0$ $\quad t = 0.423, 1.577$

MIN -0.385

MAX 0.385

$x = \int 3t^2 - 6t + 2 \; dt$

$t = 0$
$x = 0$

$x = t^3 - 3t^2 + 2t + c$ $\quad c = 0$

$x = t^3 - 3t^2 + 2t$

$\therefore 0.385$

(c) the distance travelled in the first two seconds [2]

$\int_0^2 |3t^2 - 6t + 2| \, dt = 1.54$

(d) the average velocity over the first 5 seconds [2]

$\int_0^5 (3t^2 - 6t + 2) \, dt = 60$

AVERAGE $= \frac{60}{5}$

$= 12$ m/s

9. (8 marks)

A consortium owns apartments. It discovers that if it charges $400 per week it will rent out 240 apartments. For every $5 increase in rent it will rent out 2 less apartments.

Determine

(a) Determine the number of apartments if there is a $40 increase in rent [1]

$240 - 8(2) = 224$

(b) Determine the total rent collected from all apartments if the rental is increased to $425 [2]

$425(240 - 10)$

$= 97750$

Let x be the number of $5 increases in the rental amount.

(c) Show clearly the total rental collected from all rented apartments per week will be $R(x) = 96000 + 400x - 10x^2$ [3]

R = NUMBER APARTMENTS × WEEKLY RENTAL

$= (240 - 2x)(400 + 5x)$

$= 96000 + 400x - 10x^2$

(d) Determine the number of apartments the consortium should rent out to maximise revenue and the apartment rental charged [2]

NUMBER APARTMENTS $= 240 - 20(2)$

$= 200$

RENTAL $= 400 + 20(5)$

$= \$500$